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# Systematic Approach to Gauge-Invariant Relations between Lepton Flavor Violating Processes

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## Abstract

We analyze four-lepton contact interactions that lead to lepton flavor violating processes, with violation of individual family lepton number but total lepton number conserved. In an effective Lagrangian framework, the assumption of gauge invariance leads to relations among branching ratios and cross sections of lepton flavor violating processes. In this paper, we work out how to use these relations systematically. We also study the consequences of loop-induced processes.

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## I. INTRODUCTION

The Standard Model (SM) of Particle Physics does not allow conversion between lepton flavors and thus is not able to accommodate neutrino oscillation phenomena. Therefore, the evidence of oscillations (see [1] for a recent review) makes leptons a promising sector where to look for clues about new physics. This strongly motivates the quest for other lepton flavor violating (LFV) searches; a quest that is already being carried out and that will be pushed in the near future by a variety of experiments, including the LHC and the projected neutrino factory or the linear collider.

From a theoretical point of view, LFV originates at high energies and can be conveniently described in a model independent way using an effective Lagrangian approach. We will construct our effective theory using just SM fields. Under this assumption, the minimal extension of the SM that can accommodate neutrino masses consists on adding the following dimension five operator to the SM Lagrangian [2]:

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{\alpha_{ij}}{\Lambda}(L_i H)^T (L_j H) + h.c. \tag{1}$$

This operator violates total lepton number, $L = \Sigma L_i = L_e + L_\mu + L_\tau$, by two units, and gives rise to Majorana masses for the neutrinos after the breaking of the electroweak symmetry. This dimension five operator also violates flavor, and induces rates for LFV processes suppressed by the scale $\Lambda$. In view of the measured neutrino mass splittings, this scale is presumably very high, possibly close to the Grand Unification scale. Therefore, the rates for LFV processes induced by this operator are probably too small to be observed experimentally [3].

On the other hand, there could exist violation of just the individual family lepton numbers, $\Delta L_i \neq 0$, while preserving total lepton flavor number, $\Delta L = 0$. This violation could be generated at a lower scale than $\Lambda$ in (1). In this paper we will concentrate on contact four-lepton interactions, that are unrelated to neutrino masses, and that could generate rates for the LFV processes at observable levels. In fact, this turns out to be the case in most extensions of the Standard Model, such as supersymmetry. We would like to stress that, throughout the paper, by LFV we mean violation of just the family lepton numbers; total lepton number is conserved.

We will assume that the physics underlying the effective theory respects the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. (Some of the problems that originate when

dealing with non-gauge invariant interactions at low energies have been discussed in Ref.[4].) This simple and reasonable assumption has a very profound consequence: the same operator induces several processes, that are related by gauge invariance. Therefore, one could use the power of gauge invariance to translate constraints on well studied processes into constraints on more poorly measured processes, or even not measured at all. This rationale has been applied in the past to constrain non-standard neutrino interactions [5], to rule out LFV interactions as an explanation for the LSND anomaly [6] or the atmospheric neutrino anomaly [7], and to study the prospects to observe LFV in a future muon or electron collider [8].

In this paper we will try to be more ambitious, undertaking this analysis for *all* the possible LFV effective interactions, involving only leptons and compatible with the SM gauge symmetry. To this end, we will classify all the possible operators that can induce LFV processes, and we will list different LFV processes induced by those operators, together with their present experimental bounds, if these exist. As explained above, gauge invariance relates some of these processes and their corresponding bounds. We will construct tables where these relations could be used systematically, in order to translate the bounds on the best constrained LFV processes into bounds on the worst constrained ones.

The paper is organized as follows. In the next section we work out the basis of the LFV operators. The tables are introduced in Section III, where of course we explain how to use them. In this section we also study the possibility of obtaining bounds from processes where our operators enter at one loop. We devote section IV to some comments and to present our conclusions.

## II. OPERATOR BASIS

We are interested in dimension-six operators involving four lepton fields with a current$\times$current structure. The restrictions imposed by gauge invariance on these operators constructed just with SM fields was studied by Buchmüller and Wyler in [9]. According to this reference, four classes of operators can be built involving four lepton fields, namely,

$$A_{ijkl} = (\bar{L}_i\gamma^\mu L_j)(\bar{L}_k\gamma_\mu L_l) \ , \tag{2}$$

$$B_{ijkl} = (\bar{L}_i\gamma^\mu\vec{\sigma} L_j)(\bar{L}_k\gamma_\mu\vec{\sigma} L_l) \ , \tag{3}$$

$$C_{ijkl} = (\bar{i}\gamma^\mu j)(\bar{k}\gamma_\mu l) \ , \tag{4}$$

$$D_{ijkl} = (\bar{i}\gamma^\mu j)(\bar{L}_k\gamma_\mu L_l) \ . \tag{5}$$

Here the subindices $i, j, k, l$ refer to the different lepton flavors $(e, \mu, \tau)$. For one of these flavors $i$, the left-handed lepton doublet is denoted by $L_i$, and the right-handed singlet by $i$. In eq.(3), $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ are the Pauli matrices.

There are indeed other structures that can be formed with four lepton fields, like

$$(\bar{L}_i \ j)(\bar{k} \ L_l) \ . \tag{6}$$

As discussed in [9], one can use the Fierz identity

$$(1 \pm \gamma^5)_{ab}(1 \mp \gamma^5)_{cd} = \frac{1}{2}[(1 \pm \gamma^5)\gamma^\mu]_{ad}[(1 \mp \gamma^5)\gamma_\mu]_{cb} \ , \tag{7}$$

so that eq.(6) converts trivially to $D_{kjil}$.

However, not all the operators appearing in eqs.(2-5) are independent. In fact, it turns out that the set of $A$ and the set of $B$ are linearly dependent. One has to make use of a Fierz identity for the Pauli $\sigma$ matrices,

$$\vec{\sigma}_{ab}\vec{\sigma}_{cd} = 2\delta_{ad}\delta_{cb} - \delta_{ab}\delta_{cd} \ , \tag{8}$$

and a Fierz identity for the Dirac $\gamma$ matrices,

$$[(1-\gamma^5)\gamma^\mu]_{ab}[(1-\gamma^5)\gamma_\mu]_{cd} = -\,[(1-\gamma^5)\gamma^\mu]_{ad}[(1-\gamma^5)\gamma_\mu]_{cb} \ . \tag{9}$$

With this, it is easy to prove the relation

$$\begin{pmatrix} B_{ijkl} \\ B_{ilkj} \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} A_{ijkl} \\ A_{ilkj} \end{pmatrix} \ , \tag{10}$$

valid for $j \neq l$. If $j = l$, then $B_{ijkj} = A_{ijkj}$ (no sum over $j$).

For our analysis, we have to choose one of the two sets; for convenience we will keep the set $A$ and dispose of the set $B$. Of course, in the context of a specific data set, it may prove

convenient to use $B$ instead of $A$, or even to use all of them keeping in mind the linear dependence.

In each one of the sets, $A_{ijkl}$, $C_{ijkl}$, and $D_{ijkl}$, the subindices run over lepton flavor $e, \mu, \tau$. Some operators are lepton family conserving. Even if these are beyond the Standard Model too, our aim is to study LFV, so we concentrate in lepton-family number violating operators. They may change lepton-flavor numbers by one unit $|\Delta L_i| = 1$ or two units, $|\Delta L_i| = 2$ .

In order to proceed and select a basis in the operator space, we have to take into account that not all possible combinations of flavor indices lead to independent operators. For example, trivially $A_{ijkl} = A_{klij}$. The same happens for $C$, but not for $D$. In addition, complex conjugation does not really lead to a new operator, in the sense that it is obviously the real combination $g\,O + g^*\,O^\dagger$ what appears in the Lagrangian. Finally, in the case of the $C$-type operators, Fierz rearrangements still lead to relations among different flavor combinations.

We have carefully taken into account all these arguments and reached the following conclusion: in the case of $n$ flavors, there are $n^2(n^2-1)/4$ LFV $A$-type operators, $n(n^2-1)(n+2)/8$ LFV $C$-type operators and $n(n-1)(n^2+n-1)/2$ LFV of $D$-type. This makes a total of $n(n-1)(7n^2+9n-2)/8$ LFV operators.

It is useful to enumerate the independent operators in the case of two flavors $n = 2$, that for the sake of definition we take as $e$ and $\mu$. We find three $A$-type operators, two that lead to $|\Delta L_e| = |\Delta L_\mu| = 1$, namely $A_{eee\mu}$ and $A_{e\mu\mu\mu}$, and one leading to $|\Delta L_e| = |\Delta L_\mu| = 2$, which is $A_{e\mu e\mu}$. In the $C$ sector, we also find a total of 3 operators that coincide with the order we have shown for $A$. Finally, there are five independent $D$-operators: $D_{eee\mu}$, $D_{e\mu ee}$, $D_{e\mu\mu\mu}$, and $D_{\mu\mu e\mu}$ with $|\Delta L_i| = 1$ and $D_{e\mu e\mu}$ with $|\Delta L_i| = 2$. Notice our convention that, among all possible combinations leading to an equivalent operator, in the first two indices we choose the one that has $e$ preceding $\mu$. We will follow the convention of putting the first two indices in increasing generation number, namely, $e$ will precede $\mu$ and $\tau$, and $\mu$ will precede $\tau$. When the two first indices are equal, we choose the two last indices in increasing generation number.

Since there are three flavors in nature, there are in total 18 $A$-type operators (among them 3×2=6 involving actually only two flavors), 15 $C$-type (with also 3×2=6 involving only two flavors), and 33 $D$-type (with 5×2=10 involving only two flavors). Therefore, the basis has a total of 66 LFV operators.

The effective Lagrangian describing LFV processes with total lepton number conserved is

$$\mathcal{L} = \frac{4G_F}{\sqrt{2}} \sum [a_{ijkl} A_{ijkl} + c_{ijkl} C_{ijkl} + d_{ijkl} D_{ijkl} + h.c.] \ . \tag{11}$$

We have chosen a convenient normalization that will simplify future expressions. The adimensional coefficients $a$, $c$, and $d$ are complex in general. They contain information about the high-energy scale $\Lambda$ at which the effective Lagrangian arises,

$$\frac{4G_F a_{ijkl}}{\sqrt{2}} \propto \frac{1}{\Lambda^2} \ , \tag{12}$$

and similarly for $c$'s and $d$'s.

## III. LFV PROCESSES

In the previous section we have introduced a minimal basis of operators that define the most general LFV effective Lagrangian compatible with the SM gauge symmetry, eqs.(2-5). For a given process, there are in general several LFV operators in our basis that induce it. Conversely, given one operator, it induces in general several LFV processes. This allows to relate different LFV reactions. Therefore, the experimental information available on one of them could be used to constrain other related reactions. Let us illustrate this with an example.

We consider in detail the exotic $\mu$-decay channel

$$\mu^+ \to e^+ \bar{\nu}_e \nu_e \ , \tag{13}$$

that was discussed in [10] in the context of the LSND experiment on $\bar{\nu}_e$ appearance in a $\bar{\nu}_\mu$ beam. Indeed, the LFV decay (13) could be an alternative explanation of the anomalous LSND results [11] without invoking a fourth neutrino. In fact, this is by now past history since the present bounds on (13) exclude such an alternative explanation. Independently of the LSND experiment, this decay is still of interest to us since it is related by $SU(2)_L$ gauge invariance to the better measured process $\mu \to 3e$, as discussed by Bergmann and Grossman [6], and constitutes a beautiful example of the point that we want to stress in our paper.

Let us discuss now for this particular example how to derive relations among different processes using the effective operators introduced in section II. Clearly, a gauge transformation cannot change family lepton number, therefore gauge invariance only relates processes

with the same number of leptons and anti-leptons of the same generation. For instance, the process $\mu^+ \to e^+ \bar{\nu}_e \nu_e$, that involves three leptons with $|L_e| = 1$ and one lepton with $|L_\mu| = 1$, is related by gauge invariance to $\mu^+ \to e^+ e^- e^+$ that also involves three leptons with $|L_e| = 1$ and one lepton with $|L_\mu| = 1$, but not to $\mu^+ \to e^+ \bar{\nu}_e \nu_\mu$, since it involves two leptons with $|L_e| = 1$ and two with $|L_\mu| = 1$. This classification is reflected in our tables.

It can be checked that only the operators $A_{eee\mu}$ and $D_{e\mu ee}$ contribute to the process $\mu^+ \to e^+ \bar{\nu}_e \nu_e$. Assuming that one of these operators dominates over the other, the branching ratio for this process is equal to either $|a_{eee\mu}|^2$ or $|d_{e\mu ee}|^2$, following the normalization chosen in eq.(11). (We will come back to the assumption of no fine tuned cancellations later on.) Using table I, it is straightforward to find which other processes the operators $A_{eee\mu}$ and/or $D_{e\mu ee}$ induce . For instance, it can be readily checked that $\mu^+ \to e^+ e^- e^+$ can be induced by $A_{eee\mu}$ and $D_{e\mu ee}$, and also by $C_{eee\mu}$ and $D_{eee\mu}$. The experimental information about these processes, when available, is shown in the last column of the tables. In this specific case, there is a stringent experimental bound on $\mu^+ \to e^+ e^- e^+$ [12] which leads to

$$\begin{aligned} a^2_{eee\mu} &< 0.5\ 10^{-12} \ , \\ c^2_{eee\mu} &< 2\ 10^{-12} \ , \\ d^2_{e\mu ee} &< 10^{-12} \ , \\ d^2_{eee\mu} &< 10^{-12} \ . \end{aligned} \tag{14}$$

For the sake of simplicity in the notation, we will understand here and in what follows the modulus squared of a coefficient when just the square is written. Notice that we are able to constrain each coefficient separately because of our assumption of barring fine-tuned cancellations. As a result, we find that there are other LFV processes severely limited by eq.(14). In particular the limit on the branching ratio of the decay $\mu^+ \to e^+ \bar{\nu}_e \nu_e$ coming from eq.(14) is $\mathcal{O}(10^{-12})$, and is much more restrictive than the direct experimental limit obtained in [13], which leads to $a^2_{eee\mu} \leq 9\ 10^{-4}$ , $d^2_{e\mu ee} \leq 9\ 10^{-4}$ . All this information can be read from table I. Furthermore, from the table we also find that eq.(14) leads to strong restrictions on $e^- e^+ \to e^\pm \mu^\mp$, $e^- e^- \to e^- \mu^-$, etc. We can use our arguments to bound processes like $\nu_e e \to \nu_e \mu$, which are extremely difficult to observe.

This procedure can be straightforwardly applied to other processes in the tables. Table I contains processes involving the two flavors $\mu$ and $e$, and in the next tables we list processes involving $\tau$ and $e$ (table II), and $\tau$ and $\mu$ (table III). Finally, table IV is devoted to processes

with the three flavors at work. Each table is subdivided according to the family lepton number of the particles participating in the process.

Let us summarize the use of the tables. First, one has to check the electronic, muonic and tauonic lepton number that are involved in the process we want to study and look at the corresponding subdivision in the tables. Once we have found the process we are interested in, we can read from the table the operators that contribute to that process. Barring cancellations, the experimental constraints shown in the table apply to all the operators separately, and those constraints on the operators can be translated into constraints on any other process in the table.

Notice in passing that for all the subdivisions, there is at least one process for which experimental information is available. However, this is not enough to constrain all the relevant operators. Using just the published bounds on different LFV processes, and the limits from requiring that the deviation from the SM prediction of the decay rates $\Gamma(\tau^+ \to e^+\nu_e\bar{\nu}_\tau)$ and $\Gamma(\tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau)$ is smaller than the experimental uncertainty, it is possible to constrain all the operators, except one operator of the type A, seven of the type C and seven of the type D.

Up to now, we have been considering the effective Lagrangian (11) at tree level. We will see now that some of the operators mentioned in the last paragraph can be constrained, up to an order of magnitude, using loop-induced processes. The idea is that in some of the four-lepton vertexes we can take two of the external lines, close them to form a loop, and attach an external gauge boson. (Notice that for operators with $|\Delta L_i| = 2$ no loop can be closed.) The resulting process is LFV. Take for example $\mu \to e\gamma$. It can be generated through diagrams like the one shown in Fig. 1.

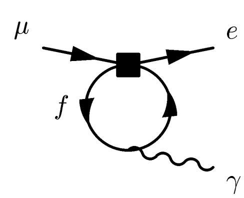


FIG. 1: Diagram for $\mu \to e\gamma$, $(f = e, \mu, \tau)$

This transition is absent at tree level and thus the loop must be finite (we have checked explicitly that the divergence of the diagram cancels). Unfortunately, the precise value of the total contribution to the decay cannot be computed, due to the presence of unknown coun-

terterms. Nevertheless, one can estimate an order of magnitude in terms of the coefficients of the four-lepton interactions [14]. Among all the possible operators in our classification, eqs.(2-5), this transition is induced by $D_{e\mu ee}$, $D_{eee\mu}$, $D_{\mu\mu e\mu}$, $D_{e\mu\mu\mu}$, $D_{e\tau\tau\mu}$, and $D_{\mu\tau\tau e}$. The resulting width is

$$\Gamma(\mu \to e\gamma) \sim \; 10^{-4}\, \alpha\, G_F^2 m_f^2 m_\mu^3 |d|^2 \;, \tag{15}$$

with $d$ representing any of the couplings of the relevant operators and $m_f$ the mass of the fermion running in the loop. The stringent experimental limit on $\mu \to e\gamma$ [12] allows to put the following order of magnitude upper bounds.

$$\begin{aligned} d^2_{eee\mu}, d^2_{e\mu ee} &< \; 10^{-5} \;\;, \\ d^2_{e\mu\mu\mu}, d^2_{\mu\mu e\mu} &< \; 10^{-9} \;\;, \\ d^2_{\tau\mu e\tau}, d^2_{e\tau\tau\mu} &< \; 10^{-11} \;\;. \end{aligned} \tag{16}$$

Having $\mu$ and $e$ as external lines, we can also attach a $Z$-boson instead of a photon and thus we will have the LFV process $Z \to \mu^\pm e^\mp$. The contribution from the loop gives

$$\Gamma(Z \to \mu^\mp e^\pm) \simeq 10^{-6} G_F^3 M_Z^7 |a|^2 ln^2 \frac{\Lambda^2}{m^2} \tag{17}$$

with $a$ standing for the relevant coupling of any of the three types. The fact that $Z$ is massive implies some important differences between the vertexes $\mu e\gamma$ and $\mu e Z$. While $\mu \to e\gamma$ is a magnetic dipole transition, for the Z the leading contribution to the amplitude is of the type

$$\bar{\mu}\gamma^\mu (F_1^V + F_1^A \gamma^5)\, e\, Z_\mu \tag{18}$$

The loop in the effective theory is calculated at $q^2 = M_Z^2$ and has the logarithm appearing in (17). Also, while for $\mu \to e\gamma$ the external muon and electron must have different quiralities, this is no longer the case for (18), so there might be now contributions from the three types of operators. To be conservative, when finding numerically our bounds, we will set $ln(\Lambda^2/m^2) = ln(\Lambda^2/M_Z^2)$ equal to 1.

The list of contributing operators to $Z \to \mu^\pm e^\mp$ is shown in table V. The LEP limits on this exotic vertex [12] lead to bounds of order 10 on the corresponding modulus square coefficients. With the table, we can compare the limits coming from $\mu \to e\gamma$ to the corresponding ones from $Z \to \mu e$. When an operator contributes to both (necessarily of the

D-type) the former are much stronger that the latter (except in the $D_{e\tau ee}$ and $D_{eee\tau}$ cases). However, for the operators contributing to $Z \to \mu e$ but not to, the limits coming from $Z$ decays are relevant, although unfortunately not very restrictive.

The same analysis can be done for $\mu - \tau$ and $e - \tau$ transitions, mediated by either $\gamma$ or $Z$. Again, the relevant information is in table V. One can draw conclusions from the table. For example one can see that magnetic dipole transitions where a $\tau$ runs in the loop lead to quite stringent bounds. The physical reason stems from the necessary helicity-flip in the loop of those transitions. The interested reader can draw other conclusions from the table.

Let us stress that the limits in table V are just up to an order of magnitude. Despite this limitation, we find them very useful, since some operators that could not be constrained using tree level processes can be constrained using loop effects. For instance this is the case for many operators that involve more than one fermion in the third generation, such as $A_{e\mu\tau\tau}, C_{e\tau\tau\tau}, D_{\tau\tau e\mu}$, etc. At the end of the day, combining results from tree level and one loop processes, we find that among the 66 independent operators that contribute to the Lagrangian, only very few remain unconstrained: three of type $C$ and two of type $D$. These are $C_{e\tau e\tau}$, $C_{\mu\tau\mu\tau}$, $C_{e\tau\mu\tau}$, $D_{e\tau e\tau}$, and $D_{\mu\tau\mu\tau}$. All of them, except $C_{e\tau\mu\tau}$, could be constrained in the future using $e^-e^-$ and $\mu^-\mu^-$ linear colliders.

## IV. FINAL COMMENTS AND CONCLUSIONS

In this section, we would like to comment on two of our assumptions: the absence of cancellations and the imposition of gauge invariance. Finally, we present the conclusions.

When discussing the decay $\mu^+ \to e^+\bar{\nu}_e\nu_e$ we did not allow for fine tuned cancellations. Even if unnatural, such cancellations may spoil some conclusions. In the example we presented before, the limits in eq.(14), coming from the $\mu \to 3e$ decay, do not hold if there is a destructive interference. For instance, one could have two operators, $A_{eee\mu}$ and $C_{eee\mu}$, that contribute in such a way that

$$|a_{eee\mu} - \frac{1}{2} c_{eee\mu}|^2 \leq \ 10^{-12} \ , \tag{19}$$

while $|a_{eee\mu}|^2 \gg \ 10^{-12}$ and $|c_{eee\mu}|^2 \gg \ 10^{-12}$. This would mean that there would be a contribution to the decay $\mu^+ \to e^+\bar{\nu}_e\nu_e$ much larger than our arguments concluded. We find this possibility very contrived, although cannot be excluded from first principles.

In the future there might be a positive signal in one (or more) of the processes listed in the tables. In this case, it will be desirable to push our type of analysis without the assumption of fine-tuned cancellations. To do that, we should work with more observables than just a branching ratio or a cross-section; we should consider for example polarization measurements. Since these involve different combinations of operators, one would be able to do a full statistical analysis and bound all the coefficients of the relevant operators.

As a final comment, we would like to stress that all the relations we have presented in the paper are based on the gauge invariance of the operators. Although we find this a very reasonable assumption, only the experiments will tell whether the relevant operators respect gauge invariance or not. A violation of these relations would be an indication for extra effects not considered in the present analysis. For instance, it could happen that for some reason dimension-six operators were forbidden, and the lowest dimension operators appear at dimension seven or eight [5]. In this case, the relations presented in this paper would not hold.

In conclusion, we have presented a systematic approach to relations between LFV processes related by the SM gauge symmetry. We have restricted ourselves to purely leptonic processes. The strategy has been to build the most general effective Lagrangian that preserves $SU(2)_L \times U(1)_Y$ and leads to purely leptonic physics. We have considered operators with energy dimension equal to six, which can be rearranged to the shape of current$\times$current. We have introduced no new fields neither right-handed neutrinos. The special shape of this effective extension has allowed to relate different LFV processes among themselves, and this has been applied to the general study of all the 66 operators involving the three flavors. We have proposed a systematic framework for studying these relations by means of the use of tables where the different LFV processes are listed. We have also discussed the implications coming from loop-induced processes. In the tables, one can review the current bounds for leptonic LFV searches.

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| | | $A$ | $C$ | $D$ | exp. bound |
|---|---|---|---|---|---|
| $2e+2\mu$ | $BR(\mu^- \to e^- \nu_e \bar{\nu}_\mu)$ | $4a^2_{e\mu e\mu}$ | - | $d^2_{e\mu e\mu}$ | $< 9 \times 10^{-4}$[13] |
| | $\tilde{P}_{\bar{M} \leftrightarrow M}$ | $a^2_{e\mu e\mu}$ | $c^2_{e\mu e\mu}$ | $d^2_{e\mu e\mu}$ | $< 3.3\ 10^{-6}$ [15] |
| | $\tilde{\sigma}(e^- e^- \to \mu^- \mu^-)$ | $12a^2_{e\mu e\mu}$ | $12c^2_{e\mu e\mu}$ | $2d^2_{e\mu e\mu}$ | |
| | $\tilde{\sigma}(\nu_e e^- \to \nu_\mu \mu^-)$ | $6a^2_{e\mu e\mu}$ | - | $\frac{1}{2}d^2_{e\mu e\mu}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\mu e^- \to \bar{\nu}_e \mu^-)$ | $2a^2_{e\mu e\mu}$ | - | $\frac{3}{2}d^2_{e\mu e\mu}$ | $< 9\ 10^{-3}$ [16] |
| $3e+1\mu$ | $BR(\mu^- \to e^- \bar{\nu}_e \nu_e)$ | $a^2_{eee\mu}$ | - | $d^2_{eee\mu}$ | $< 9\ 10^{-4}$ [13] |
| | $BR(\mu^- \to e^- e^+ e^-)$ | $2a^2_{eee\mu}$ | $2c^2_{eee\mu}$ | $d^2_{eee\mu}, d^2_{e\mu ee}$ | $<\ 10^{-12}$ [17] |
| | $\tilde{\sigma}(e^+ e^- \to e^\pm \mu^\mp)$ | $4a^2_{eee\mu}$ | $4c^2_{eee\mu}$ | $4d^2_{eee\mu}, 4d^2_{e\mu ee}$ | $< 2.3\ 10^{-4}$ [18] |
| | $\tilde{\sigma}(e^- e^- \to e^- \mu^-)$ | $6a^2_{eee\mu}$ | $6c^2_{eee\mu}$ | $2d^2_{eee\mu}, 2d^2_{e\mu ee}$ | |
| | $\tilde{\sigma}(\nu_e e^- \to \nu_e \mu^-)$ | $\frac{3}{2}a^2_{eee\mu}$ | - | $\frac{1}{2}d^2_{eee\mu}$ | |
| | $\tilde{\sigma}(\bar{\nu}_e e^- \to \bar{\nu}_e \mu^-)$ | $\frac{1}{2}a^2_{eee\mu}$ | - | $\frac{3}{2}d^2_{eee\mu}$ | |
| $1e+3\mu$ | $BR(\mu \to e \bar{\nu}_\mu \nu_\mu)$ | $a^2_{e\mu\mu\mu}$ | - | $d^2_{e\mu\mu\mu}$ | |
| | $\tilde{\sigma}(\mu^+ \mu^- \to \mu^\pm e^\mp)$ | $4a^2_{e\mu\mu\mu}$ | $4c^2_{e\mu\mu\mu}$ | $4d^2_{e\mu\mu\mu}, 4d^2_{\mu\mu e\mu}$ | |
| | $\tilde{\sigma}(\mu^- \mu^- \to e^- \mu^-)$ | $6a^2_{e\mu\mu\mu}$ | $6c^2_{e\mu\mu\mu}$ | $2d^2_{e\mu\mu\mu}, 2d^2_{\mu\mu e\mu}$ | |
| | $\tilde{\sigma}(\nu_\mu e^- \to \nu_\mu \mu^-)$ | $\frac{3}{2}a^2_{e\mu\mu\mu}$ | - | $\frac{1}{2}d^2_{e\mu\mu\mu}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu \mu^-)$ | $\frac{1}{2}a^2_{e\mu\mu\mu}$ | - | $\frac{3}{2}d^2_{e\mu\mu\mu}$ | $< 9\ 10^{-3}$ [16] |

TABLE I: LFV processes involving electron and muon flavors. For the coefficients, it should be understood modulus squared where just the square is written (for instance, $a^2_{e\mu e\mu}$ stands for $|a_{e\mu e\mu}|^2$). $\tilde{P}_{M-\bar{M}}$ represents the probability of the transition muonium-antimuonium normalized, for the sake of conciseness in the presentation, to $P_0 = 64e^{12}\left[m_e^3/G_F m_\mu^5\right]^2 = 2.56\ 10^{-5}$ with $e$ the electric charge. Therefore, $\tilde{P}_{M-\bar{M}} = P_{M-\bar{M}}/P_0$. On the other hand, $\tilde{\sigma}(X+Y \to X'+Y')$ is the cross section of the process $X+Y \to X'+Y'$ normalized to $\sigma_0 = G_F^2 s/3\pi$, *i.e.* $\tilde{\sigma}(X+Y \to X'+Y') = \sigma(X+Y \to X'+Y')/\sigma_0$. In the tables, near the threshold for heavy lepton production in electron-(anti)neutrino scattering, one has to substitute

| | $\nu e^- \to \nu l^-$ | $\bar{\nu} e^- \to \bar{\nu} l^-$ |
|---|---|---|
| $a^2$ | $a^2(1-\frac{m_l^2}{s})^2$ | $a^2(1-\frac{m_l^2}{s})^2(1+\frac{m_l^2}{2s})$ |
| $d^2$ | $d^2(1-\frac{m_l^2}{s})^2(1+\frac{m_l^2}{2s})$ | $d^2(1-\frac{m_l^2}{s})^2$ |

| | | $A$ | $C$ | $D$ | exp. bound |
|---|---|---|---|---|---|
| $2e+2\tau$ | $\tilde{B}(\tau^- \to e^- \bar{\nu}_\tau \nu_e)$ | $4a^2_{e\tau e\tau}$ | - | $d^2_{e\tau e\tau}$ | $< 8\ 10^{-3}$ [19] |
| | $\tilde{\sigma}(e^- e^- \to \tau^- \tau^-)$ | $12a^2_{e\tau e\tau}$ | $12c^2_{e\tau e\tau}$ | $2d^2_{e\tau e\tau}$ | |
| | $\tilde{\sigma}(\nu_e e^- \to \nu_\tau \tau^-)$ | $6a^2_{e\tau e\tau}$ | - | $\frac{1}{2}d^2_{e\tau e\tau}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\tau e^- \to \bar{\nu}_e \tau^-)$ | $2a^2_{e\tau e\tau}$ | - | $\frac{3}{2}d^2_{e\tau e\tau}$ | |
| $3e+1\tau$ | $\tilde{B}(\tau^- \to e^- \bar{\nu}_e \nu_e)$ | $a^2_{eee\tau}$ | - | $d^2_{e\tau ee}$ | $< 8\ 10^{-3}$ [19] |
| | $\tilde{B}(\tau^- \to e^- e^+ e^-)$ | $2a^2_{eee\tau}$ | $2c^2_{eee\tau}$ | $d^2_{e\tau ee}, d^2_{eee\tau}$ | $< 1.6\ 10^{-5}$ [20] |
| | $\tilde{\sigma}(e^- e^+ \to e^\pm \tau^\mp)$ | $4a^2_{eee\tau}$ | $4c^2_{eee\tau}$ | $4d^2_{e\tau ee}, 4d^2_{eee\tau}$ | $< 1.3\ 10^{-3}$ [18] |
| | $\tilde{\sigma}(e^- e^- \to e^- \tau^-)$ | $6a^2_{eee\tau}$ | $6c^2_{eee\tau}$ | $2d^2_{e\tau ee}, 2d^2_{eee\tau}$ | |
| | $\tilde{\sigma}(\nu_e e^- \to \nu_e \tau^-)$ | $\frac{3}{2}a^2_{eee\tau}$ | - | $\frac{1}{2}d^2_{e\tau ee}$ | |
| | $\tilde{\sigma}(\bar{\nu}_e e^- \to \bar{\nu}_e \tau^-)$ | $\frac{1}{2}a^2_{eee\tau}$ | - | $\frac{3}{2}d^2_{e\tau ee}$ | |
| $1e+3\tau$ | $\tilde{B}(\tau^- \to e^- \bar{\nu}_\tau \nu_\tau)$ | $a^2_{e\tau\tau\tau}$ | - | $d^2_{e\tau\tau\tau}$ | $< 8\ 10^{-3}$ [19] |
| | $\tilde{\sigma}(\nu_\tau e^- \to \nu_\tau \tau^-)$ | $\frac{3}{2}a^2_{e\tau\tau\tau}$ | - | $\frac{1}{2}d^2_{e\tau\tau\tau}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\tau e^- \to \bar{\nu}_\tau \tau^-)$ | $\frac{1}{2}a^2_{e\tau\tau\tau}$ | - | $\frac{3}{2}d^2_{e\tau\tau\tau}$ | |

TABLE II: LFV processes involving electron and tau flavors. $\tilde{B}(\tau \to ll'l'')$ represents the branching ratio of the corresponding rare tau decay, normalized to $BR(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$, *i.e.* $\tilde{B}(\tau \to ll'l'') = BR(\tau \to lll'')/BR(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$. For further explanations in the notation, see table I .

| | | $A$ | $C$ | $D$ | exp. bound |
|---|---|---|---|---|---|
| $2\mu+2\tau$ | $\tilde{B}(\tau^- \to \mu^- \bar{\nu}_\tau \nu_\mu)$ | $4a^2_{\mu\tau\mu\tau}$ | - | $d^2_{\mu\tau\mu\tau}$ | $< 9\ 10^{-3}$ [19] |
| | $\tilde{\sigma}(\mu^- \mu^- \to \tau^- \tau^-)$ | $12a^2_{\mu\tau\mu\tau}$ | $12c^2_{\mu\tau\mu\tau}$ | $2d^2_{\mu\tau\mu\tau}$ | |
| $3\mu+1\tau$ | $\tilde{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\mu)$ | $a^2_{\mu\mu\mu\tau}$ | - | $d^2_{\mu\tau\mu\mu}$ | $< 9\ 10^{-3}$ [19] |
| | $\tilde{B}(\tau^- \to \mu^- \mu^+ \mu^-)$ | $2a^2_{\mu\mu\mu\tau}$ | $2c^2_{\mu\mu\mu\tau}$ | $d^2_{\mu\tau\mu\mu}, d^2_{\mu\mu\mu\tau}$ | $< 1.1\ 10^{-5}$ [20] |
| | $\tilde{\sigma}(\mu^- \mu^+ \to \tau^\pm \mu^\mp)$ | $4a^2_{\mu\mu\mu\tau}$ | $4c^2_{\mu\mu\mu\tau}$ | $4d^2_{\mu\tau\mu\mu}, 4d^2_{\mu\mu\mu\tau}$ | |
| | $\tilde{\sigma}(\mu^- \mu^- \to \mu^- \tau^-)$ | $6a^2_{\mu\mu\mu\tau}$ | $6c^2_{\mu\mu\mu\tau}$ | $2d^2_{\mu\tau\mu\mu}, 2d^2_{\mu\mu\mu\tau}$ | |
| $1\mu+3\tau$ | $\tilde{B}(\tau^- \to \mu^- \bar{\nu}_\tau \nu_\tau)$ | $a^2_{\mu\tau\tau\tau}$ | - | $d^2_{\mu\tau\tau\tau}$ | $< 9\ 10^{-3}$ [19] |

TABLE III: LFV processes involving muon and tau flavors. See explanations in tables I and II.

| | | $A$ | $C$ | $D$ | exp. bound |
|---|---|---|---|---|---|
| $2e$ | $BR(\mu^- \to e^- \bar{\nu}_e \nu_\tau)$ | $a^2_{e\mu\tau e}$ | - | $d^2_{e\mu\tau e}$ | |
| $\Delta L_e = 0$ | $\tilde{B}(\tau^- \to \mu^- \bar{\nu}_e \nu_e)$ | $a^2_{ee\mu\tau}$ | | $d^2_{\mu\tau ee}$ | $< 9\ 10^{-3}$ [19] |
| | $\tilde{B}(\tau^- \to e^- \bar{\nu}_e \nu_\mu)$ | $a^2_{e\mu\tau e}$ | - | $d^2_{e\tau\mu e}$ | $< 8\ 10^{-3}$ [19] |
| | $\tilde{B}(\tau^- \to \mu^- e^+ e^-)$ | $a^2_{ee\mu\tau}, a^2_{e\mu\tau e}$ | $c^2_{ee\mu\tau}$ | $d^2_{\mu\tau ee}, d^2_{e\tau\mu e}, d^2_{ee\mu\tau}, d^2_{e\mu\tau e}$ | $< \ 10^{-5}$ [20] |
| | $\tilde{\sigma}(e^- e^+ \to \mu^\pm \tau^\mp)$ | $a^2_{ee\mu\tau}, a^2_{e\mu\tau e}$ | $c^2_{ee\mu\tau}$ | $d^2_{\mu\tau ee}, d^2_{e\tau\mu e}, d^2_{ee\mu\tau}, d^2_{e\mu\tau e}$ | $< 1.6\ 10^{-3}$ [18] |
| | $\tilde{\sigma}(\nu_\tau e^- \to \nu_e \mu^-)$ | $\frac{3}{2} a^2_{e\mu\tau e}$ | - | $\frac{1}{2} d^2_{e\mu\tau e}$ | |
| | $\tilde{\sigma}(\bar{\nu}_e e^- \to \bar{\nu}_\tau \mu^-)$ | $\frac{1}{2} a^2_{e\mu\tau e}$ | - | $\frac{3}{2} d^2_{e\mu\tau e}$ | |
| | $\tilde{\sigma}(\nu_\mu e^- \to \nu_e \tau^-)$ | $\frac{3}{2} a^2_{e\mu\tau e}$ | - | $\frac{1}{2} d^2_{e\tau\mu e}$ | |
| | $\tilde{\sigma}(\bar{\nu}_e e^- \to \bar{\nu}_\mu \tau^-)$ | $\frac{1}{2} a^2_{e\mu\tau e}$ | - | $\frac{3}{2} d^2_{e\tau\mu e}$ | |
| $2e$ | $BR(\mu^- \to e^- \bar{\nu}_\tau \nu_e)$ | $a^2_{e\mu e\tau}$ | - | $d^2_{e\mu e\tau}$ | |
| $\Delta L_e = 2$ | $\tilde{B}(\tau^- \to e^- \bar{\nu}_\mu \nu_e)$ | $a^2_{e\mu e\tau}$ | - | $d^2_{e\tau e\mu}$ | $< 8\ 10^{-3}$ [19] |
| | $\tilde{B}(\tau^- \to e^- \mu^+ e^-)$ | $2a^2_{e\mu e\tau}$ | $2c^2_{e\mu e\tau}$ | $d^2_{e\mu e\tau}, d^2_{e\tau e\mu}$ | $< 9\ 10^{-6}$ [20] |
| | $\tilde{\sigma}(e^- e^- \to \mu^- \tau^-)$ | $6a^2_{e\mu e\tau}$ | $6c^2_{e\mu e\tau}$ | $2d^2_{e\mu e\tau}, d^2_{e\tau e\mu}$ | |
| | $\tilde{\sigma}(\nu_e e^- \to \nu_\tau \mu^-)$ | $\frac{3}{2} a^2_{e\mu e\tau}$ | - | $\frac{1}{2} d^2_{e\mu e\tau}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\tau e^- \to \bar{\nu}_e \mu^-)$ | $\frac{1}{2} a^2_{e\mu e\tau}$ | - | $\frac{3}{2} d^2_{e\mu e\tau}$ | |
| | $\tilde{\sigma}(\nu_e e^- \to \nu_\mu \tau^-)$ | $\frac{3}{2} a^2_{e\mu e\tau}$ | - | $\frac{1}{2} d^2_{e\tau e\mu}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\mu e^- \to \bar{\nu}_e \tau^-)$ | $\frac{1}{2} a^2_{e\mu e\tau}$ | - | $\frac{3}{2} d^2_{e\tau e\mu}$ | |
| $2\mu$ | $BR(\mu^- \to e^- \bar{\nu}_\mu \nu_\tau)$ | $a^2_{e\mu\mu\tau}$ | - | $d^2_{e\mu\mu\tau}$ | |
| $\Delta L_\mu = 0$ | $\tilde{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_e)$ | $a^2_{e\mu\mu\tau}$ | - | $d^2_{\mu\tau e\mu}$ | $< 9\ 10^{-3}$ [19] |
| | $\tilde{B}(\tau^- \to e^- \bar{\nu}_\mu \nu_\mu)$ | $a^2_{e\tau\mu\mu}$ | - | $d^2_{e\tau\mu\mu}$ | $< 8\ 10^{-3}$ [19] |
| | $\tilde{B}(\tau^- \to e^- \mu^+ \mu^-)$ | $a^2_{e\tau\mu\mu}, a^2_{e\mu\mu\tau}$ | $c^2_{e\mu\mu\tau}$ | $d^2_{e\tau\mu\mu}, d^2_{e\mu\mu\tau}, d^2_{\mu\mu e\tau}, d^2_{\mu\tau e\mu}$ | $< \ 10^{-5}$ [20] |
| | $\tilde{\sigma}(\mu^- \mu^+ \to e^\pm \tau^\mp)$ | $a^2_{e\tau\mu\mu}, a^2_{e\mu\mu\tau}$ | $c^2_{e\mu\mu\tau}$ | $d^2_{e\tau\mu\mu}, d^2_{e\mu\mu\tau}, d^2_{\mu\mu e\tau}, d^2_{\mu\tau e\mu}$ | |
| | $\tilde{\sigma}(\nu_\mu e^- \to \nu_\tau \mu^-)$ | $\frac{3}{2} a^2_{e\mu\mu\tau}$ | - | $\frac{1}{2} d^2_{e\mu\mu\tau}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\tau e^- \to \bar{\nu}_\mu \mu^-)$ | $\frac{1}{2} a^2_{e\mu\mu\tau}$ | - | $\frac{3}{2} d^2_{e\mu\mu\tau}$ | |
| | $\tilde{\sigma}(\nu_\mu e^- \to \nu_\mu \tau^-)$ | $\frac{3}{2} a^2_{e\tau\mu\mu}$ | - | $\frac{1}{2} d^2_{e\tau\mu\mu}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu \tau^-)$ | $\frac{1}{2} a^2_{e\tau\mu\mu}$ | - | $\frac{3}{2} d^2_{e\tau\mu\mu}$ | |
| $2\mu$ | $BR(\mu^- \to e^- \bar{\nu}_\mu \nu_\tau)$ | $a^2_{e\mu\tau\mu}$ | - | $d^2_{e\mu\tau\mu}$ | |
| $\Delta L_\mu = 2$ | $\tilde{B}(\tau^- \to \mu^- \bar{\nu}_e \nu_\mu)$ | $a^2_{e\mu\tau\mu}$ | - | $d^2_{\mu\tau\mu e}$ | |
| | $\tilde{B}(\tau^- \to \mu^- e^+ \mu^-)$ | $2a^2_{e\mu\tau\mu}$ | $2c^2_{e\mu\tau\mu}$ | $d^2_{e\mu\tau\mu}, d^2_{\mu\tau\mu e}$ | $< 9\ 10^{-6}$ [20] |
| | $\tilde{\sigma}(\mu^- \mu^- \to e^- \tau^-)$ | $6a^2_{e\mu\tau\mu}$ | $6c^2_{e\mu\tau\mu}$ | $2d^2_{e\mu\tau\mu}, 2d^2_{\mu\tau\mu e}$ | |
| | $\tilde{\sigma}(\nu_\tau e^- \to \nu_\mu \mu^-)$ | $\frac{3}{2} a^2_{e\mu\tau\mu}$ | - | $\frac{1}{2} d^2_{e\mu\tau\mu}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\mu e^- \to \bar{\nu}_\tau \mu^-)$ | $\frac{1}{2} a^2_{e\mu\tau\mu}$ | - | $\frac{3}{2} d^2_{e\mu\tau\mu}$ | $< 9\ 10^{-3}$ [16] |
| $2\tau$ | $\tilde{B}(\mu^- \to e^- \bar{\nu}_\tau \nu_\tau)$ | $a^2_{e\mu\tau\tau}$ | - | $d^2_{e\mu\tau\tau}$ | |
| $\Delta L_\mu = 0$ | $\tilde{B}(\tau^- \to \mu^- \bar{\nu}_e \nu_\tau)$ | $a^2_{e\tau\tau\mu}$ | - | $d^2_{\mu\tau\tau e}$ | $< 9\ 10^{-3}$ [19] |
| | $\tilde{B}(\tau^- \to e^- \bar{\nu}_\mu \nu_\tau)$ | $a^2_{e\tau\tau\mu}$ | - | $d^2_{e\tau\tau\mu}$ | $< 8\ 10^{-3}$ [19] |
| | $\tilde{\sigma}(\nu_\tau e^- \to \nu_\tau \mu^-)$ | $\frac{3}{2} a^2_{e\mu\tau\tau}$ | - | $\frac{1}{2} d^2_{e\mu\tau\tau}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\tau e^- \to \bar{\nu}_\tau \mu^-)$ | $\frac{1}{2} a^2_{e\mu\tau\tau}$ | - | $\frac{3}{2} d^2_{e\mu\tau\tau}$ | |
| | $\tilde{\sigma}(\nu_\tau e^- \to \nu_\mu \tau^-)$ | $\frac{3}{2} a^2_{e\tau\tau\mu}$ | - | $\frac{1}{2} d^2_{e\tau\tau\mu}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\mu e^- \to \bar{\nu}_\tau \tau^-)$ | $\frac{1}{2} a^2_{e\tau\tau\mu}$ | - | $\frac{3}{2} d^2_{e\tau\tau\mu}$ | |
| $2\tau$ | $\tilde{B}(\tau^- \to e^- \bar{\nu}_\tau \nu_\mu)$ | $a^2_{e\tau\mu\tau}$ | - | $d^2_{e\tau\mu\tau}$ | $< 8\ 10^{-3}$ [19] |
| $\Delta L_\tau = 2$ | $\tilde{B}(\tau^- \to \mu^- \bar{\nu}_\tau \nu_e)$ | $a^2_{e\tau\mu\tau}$ | - | $d^2_{\mu\tau e\tau}$ | $< 9\ 10^{-3}$ [19] |
| | $\tilde{\sigma}(\nu_\mu e^- \to \nu_\tau \tau^-)$ | $\frac{3}{2} a^2_{e\tau\mu\tau}$ | - | $\frac{1}{2} d^2_{e\tau\mu\tau}$ | |
| | $\tilde{\sigma}(\bar{\nu}_\tau e^- \to \bar{\nu}_\mu \tau^-)$ | $\frac{1}{2} a^2_{e\tau\mu\tau}$ | - | $\frac{3}{2} d^2_{e\tau\mu\tau}$ | |

TABLE IV: LFV processes involving electron, muon and tau flavors.. See explanations in tables I

| | e in loop | $\mu$ in loop | $\tau$ in loop |
|---|---|---|---|
| $\mu \to e\gamma$ | $d^2_{eee\mu}, d^2_{e\mu ee} <\ 10^{-5}$ | $d^2_{e\mu\mu\mu}, d^2_{\mu\mu e\mu} <\ 10^{-9}$ | $d^2_{e\tau\tau\mu}, d^2_{\mu\tau\tau e} <\ 10^{-11}$ |
| $\tau \to e\gamma$ | $d^2_{eee\tau}, d^2_{e\tau ee} <\ 10^{4}$ | $d^2_{e\mu\mu\tau}, d^2_{\mu\tau e\mu} <\ 10^{-1}$ | $d^2_{e\tau\tau\tau}, d^2_{\tau\tau e\tau} <\ 10^{-3}$ |
| $\tau \to \mu\gamma$ | $d^2_{e\mu\tau e}, d^2_{e\tau\mu e} <\ 10^{4}$ | $d^2_{\mu\mu\mu\tau}, d^2_{\mu\mu\mu\tau} <\ 10^{-1}$ | $d^2_{\mu\tau\tau\tau}, d^2_{\tau\tau\mu\tau} <\ 10^{-3}$ |
| $Z \to e^\pm\mu^\mp$ | $a^2_{eee\mu}, c^2_{eee\mu}$, $d^2_{eee\mu}, d^2_{e\mu ee} < 10$ | $a^2_{e\mu\mu\mu}, c^2_{e\mu\mu\mu}$, $d^2_{e\mu\mu\mu}, d^2_{\mu\mu e\mu} < 10$ | $a^2_{e\tau\tau\mu}, a^2_{e\mu\tau\tau}, c^2_{e\mu\tau\tau}$, $d^2_{\tau\tau e\mu}, d^2_{e\mu\tau\tau} < 10$ |
| $Z \to \tau^\pm e^\mp$ | $a^2_{eee\tau}, c^2_{eee\tau}$, $d^2_{eee\tau}, d^2_{e\tau ee} < 100$ | $a^2_{e\mu\mu\tau}, a^2_{e\tau\mu\mu}, c^2_{e\tau\tau\mu}$, $d^2_{e\tau\mu\mu}, d^2_{\mu\mu e\tau} < 100$ | $a^2_{e\tau\tau\tau}, c^2_{e\tau\tau\tau}$, $d^2_{e\tau\tau\tau}, d^2_{\tau\tau e\tau} < 100$ |
| $Z \to \mu^\pm\tau^\mp$ | $a^2_{e\mu\tau e}, a^2_{ee\mu\tau}, c^2_{ee\mu\tau}$ $d^2_{\mu\tau ee}, d^2_{ee\mu\tau} < 100$ | $a^2_{\mu\mu\mu\tau}, c^2_{\mu\mu\mu\tau}$, $d^2_{\mu\mu\mu\tau}, d^2_{\mu\mu\mu\tau} < 100$ | $a^2_{\mu\tau\tau\tau}, c^2_{\mu\tau\tau\tau}$, $d^2_{\mu\tau\tau\tau}, d^2_{\tau\tau\mu\tau} < 100$ |

TABLE V: Operators contributing to LFV processes via one-loop contributions; the resulting bounds are nothing but an order of magnitude.